

Structure of the Nucleon from Lattice QCD

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The 4th Berkeley School
“Collective Dynamics in High-Energy Collisions”
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1 Introduction

2 Nucleon Structure from Lattice QCD

- Electromagnetic Form Factors
- Axial Form Factors
- Quark momentum and angular momentum

3 Summary

Ab-initio Calculation of Nucleon Structure

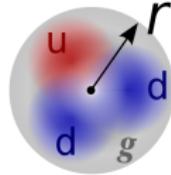
No satisfactory theoretical understanding of nucleon structure

Existing and coming experimental data:

- Proton and neutron size and charge distribution
- Partonic picture of nucleons
- Spin puzzle: part of nucleon spin must be carried by gluons and/or quark orbital momentum

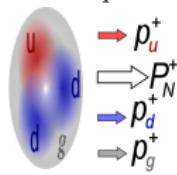
Radius

$$\langle r_E^2 \rangle^{p-n} \approx 0.85 \text{ fm}^2$$



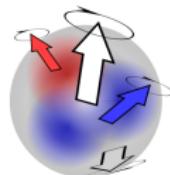
Momentum

$$P_N^+ = p_q^+ + p_g^+$$



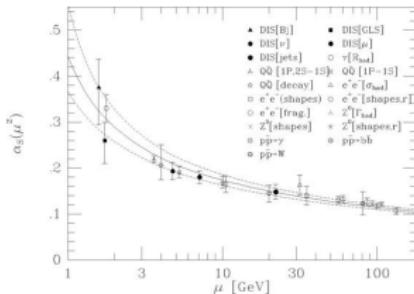
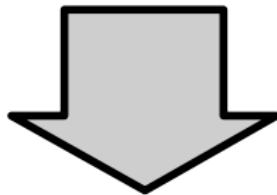
Spin

$$\frac{1}{2} = L_q + S_q + J_g$$



QCD in the Low-Energy Regime

- gauge bosons are charged \Rightarrow antiscreening:
quark-gluon coupling grows at low energy
(i.e. long distance $\gtrsim 1$ fm)
- at low energy $\mu \sim m_{p,n}$, QCD is non-perturbative:
quark masses \sim few MeV \Rightarrow “dressed” by gluons
 ≈ 300 MeV
- confinement: no free quarks exist in vacuum



Need special methods to study
QCD bound states

QCD on a Lattice: Numerical Feynman Integration

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{O}[U, \psi, \bar{\psi}] e^{-S[U, \psi, \bar{\psi}]} \rightarrow \frac{1}{N} \sum^N \tilde{\mathcal{O}}[U],$$

$$\left(\text{with Probability}[U] \sim \prod_{f=u,d,s} \det [\not{D}[U] + m_f] e^{-S_g[U]} \right)$$

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- Euclidean QFT: $\begin{cases} x^0 \equiv t & \rightarrow -ix_4 \equiv -i\tau \\ p^0 \equiv E & \rightarrow ip_4 \\ \langle N(t) \bar{N}(0) \rangle & \rightarrow e^{-E\tau} \end{cases}$

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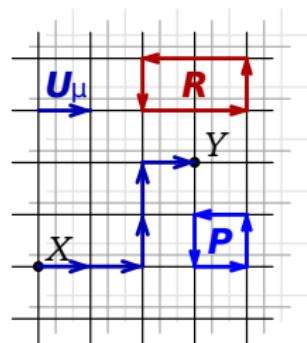
- Discretize fields on a space-time grid:

$$A_\mu^a(x) \rightarrow U_{x,\mu} = \mathcal{P}e^{-i \int_x^{x+\hat{\mu}} dx \cdot (A^a \frac{\lambda^a}{2})}$$

$$(D_\mu \varphi)_x \rightarrow \frac{1}{a} (U_{x,\mu} \varphi_{x+\hat{\mu}} - \varphi_x)$$

$$S_g[A_\mu] \sim (F_{\mu\nu}^a)^2 \rightarrow A \text{Tr}(\boxed{P}) + B \text{Tr}(\boxed{R})$$

- Fermions on a lattice: pick two from no “doublers”; chiral symmetry; simulation speed.



QCD on a Lattice: Numerical Feynman Integration

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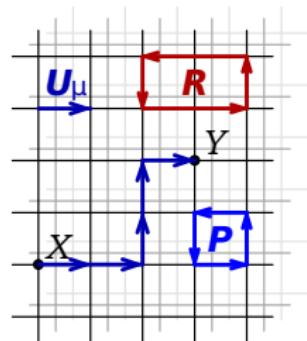
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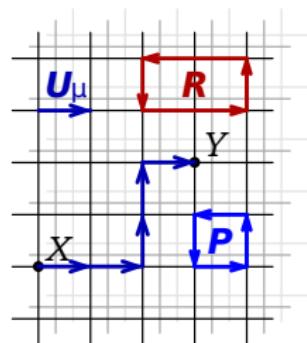
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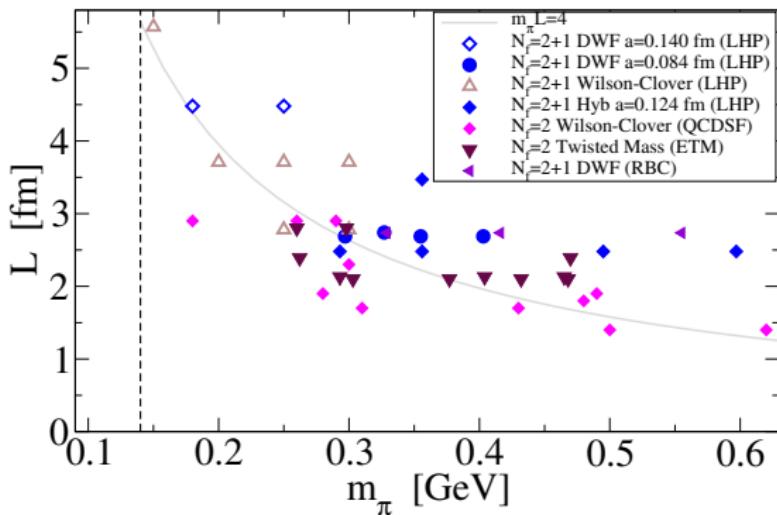
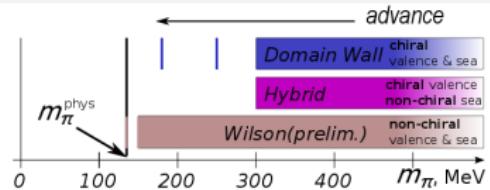
- Fermions on a lattice: pick two from no “doublers”; chiral symmetry; simulation speed.
- Tune $(\alpha_S^{\text{lat}}, am_{ud}, am_s)$ to reproduce e.g. (m_π, m_K, m_Ω) .
- **PROFIT!!!**



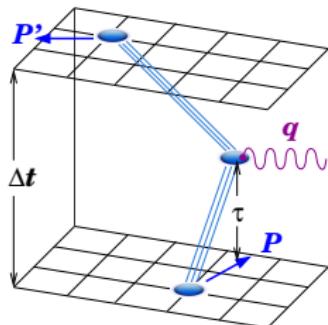
Lattice QCD is a Hard Problem

Solving QCD numerically is hard because

- light quarks are expensive: cost $\sim \frac{1}{m_\pi}$
- need large physical size of the box $L \gtrsim \frac{4}{m_\pi}$
- have to take continuum limit $a \rightarrow 0$, $L_{\text{lat}} = \frac{L}{a} \rightarrow \infty$
- chiral symmetry is expensive to preserve on a lattice



Hadron Matrix Elements



Extract $\langle P' | \mathcal{O} | P \rangle$ from 3-pt correlators

$$\sum_{\vec{x}, \vec{y}} e^{-i\vec{P}'\vec{x} + i\vec{q}\vec{y}} \langle N(\Delta t, \vec{x}) \mathcal{O}(\tau, \vec{y}) \bar{N}(0) \rangle$$

where for the proton

$$N_\alpha = \epsilon^{abc} u_\alpha^a [(u^b)^T C \gamma_5 d^c]$$

All QCD states are present:

$$\langle N(\Delta t) \mathcal{O}(\tau) \bar{N}(0) \rangle \sim \sum_{m,n} Z_m \cdot e^{-E_m(\Delta t - \tau)} \cdot \mathcal{O}_{mn} \cdot e^{-E_n \tau} \cdot Z_n^\dagger$$

Excited states can lead to systematic bias in m.e. :

$$\bar{N}_{\text{lat}} |\Omega\rangle = |N\rangle + C|X\rangle, \quad \Delta M = M_X - M_N,$$

$$\langle N | \mathcal{O} | N \rangle_{\text{lat}} \cong \langle N | \mathcal{O} | N \rangle + |C|^2 \langle X | \mathcal{O} | X \rangle e^{-\Delta M \cdot \Delta t} + \text{"tails"}$$

$$\text{Signal / noise} \sim e^{-(M_N - \frac{3}{2}m_\pi) \cdot \Delta t}$$

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Electromagnetic Form Factors

$$\langle P' | \bar{q} \gamma^\mu q | P \rangle = \bar{U}(P') \left[F_1^q(Q^2) \gamma^\mu + F_2^q(Q^2) \frac{i \sigma^{\mu\nu} q_\nu}{2M_N} \right] U(P),$$

where $q = P' - P$ and $Q^2 = -q^2$

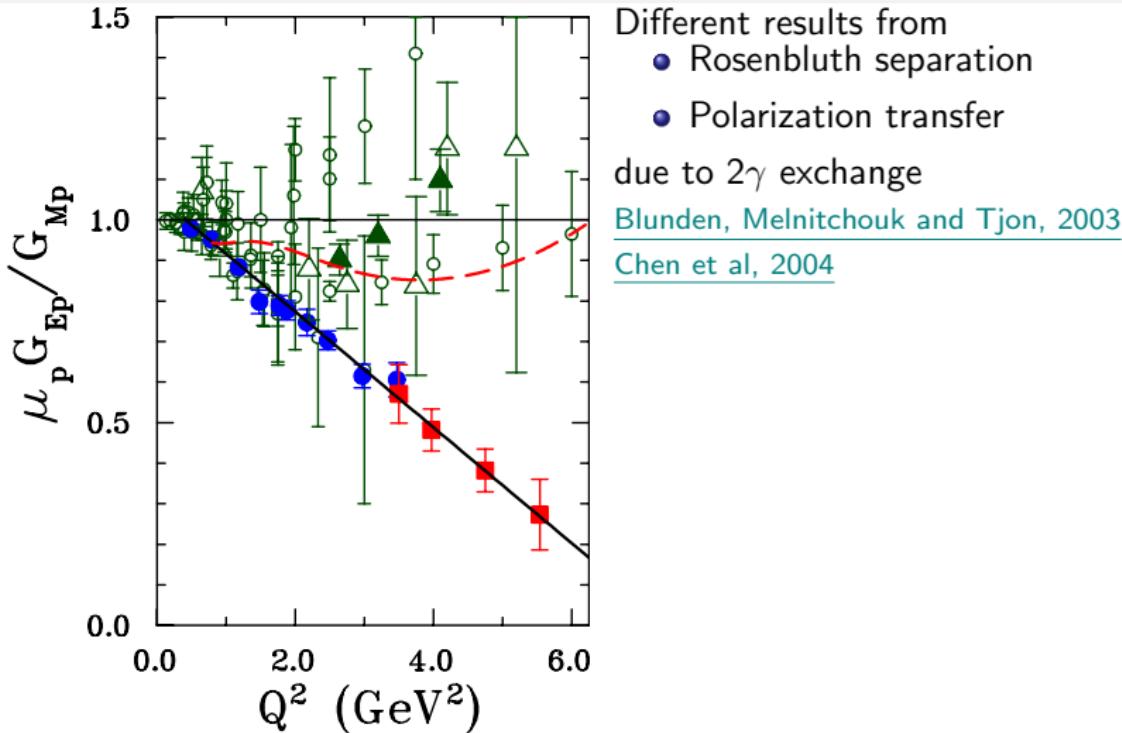
Sachs form factors

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), & G_E(0) &= Q, \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2), & G_M(0) &= \mu. \end{aligned}$$

“Radii”

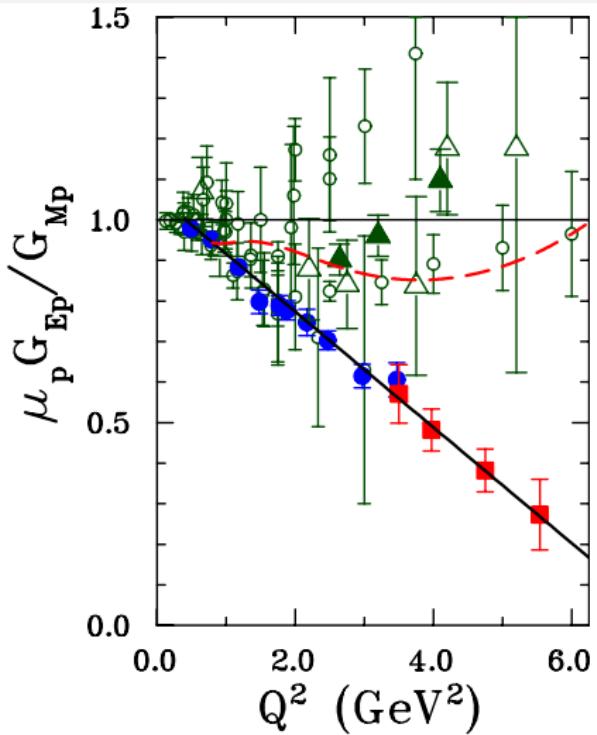
$$F(Q^2) \approx F(0) \left(1 - \frac{1}{6} \langle \mathbf{r}^2 \rangle \cdot Q^2 + O(Q^4) \right)$$

Proton Form Factors



[M. Vanderhaeghen, Nucl.Phys.A805:210\(2008\)](#)

Proton Form Factors



M. Vanderhaeghen, Nucl.Phys.A805:210(2008)

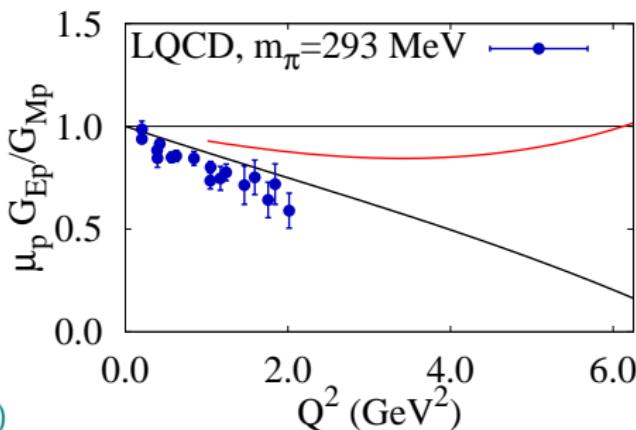
Different results from
 • Rosenbluth separation
 • Polarization transfer

due to 2γ exchange

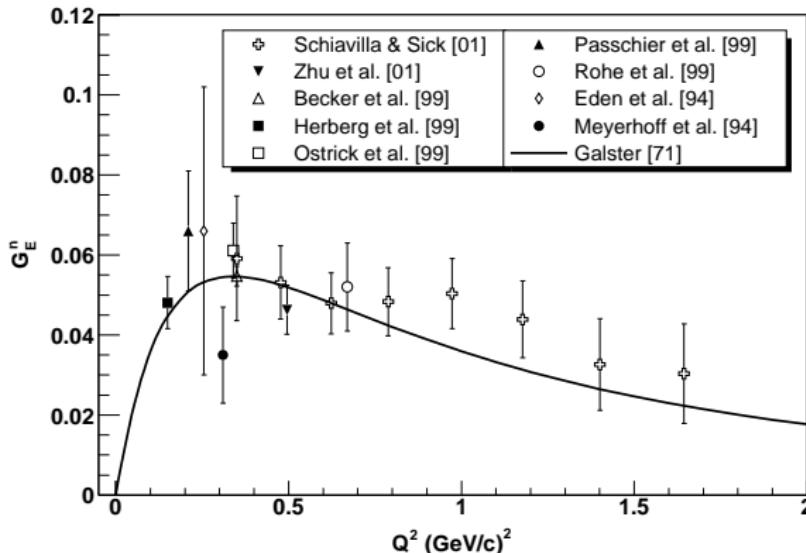
[Blunden, Melnitchouk and Tjon, 2003](#)

[Chen et al, 2004](#)

Lattice (systematics unclear!):



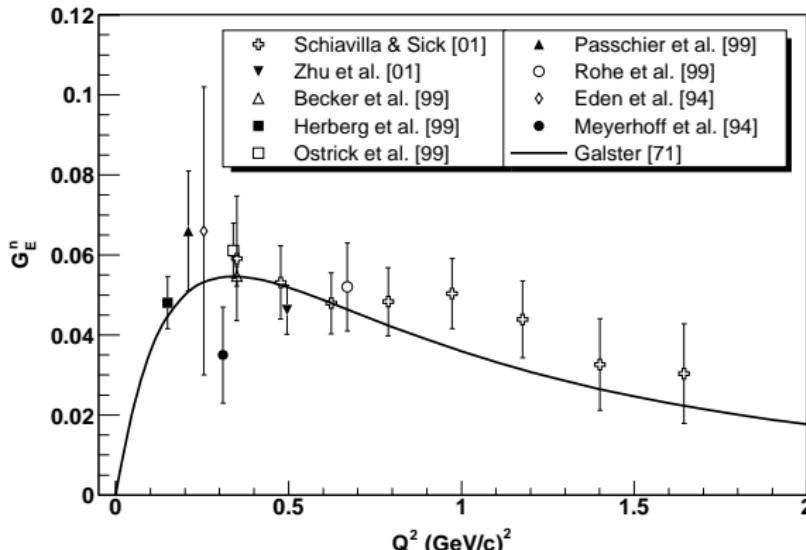
Neutron Electric Form Factor



H. Gao, Int.J.Mod.Phys.E12:1 (2003)

- deuterium targets
 - thermal neutron scattering:
 $\langle r_{E,n}^2 \rangle = -0.113(3)(4) \text{ fm}^2$
- PRL 74, 2427 (1995)

Neutron Electric Form Factor



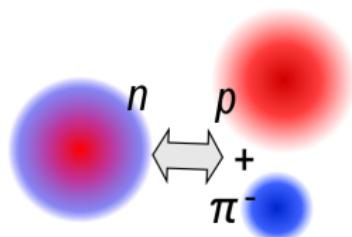
$$\langle r_{E,n}^2 \rangle < 0:$$

(+)

core

(−)

surface



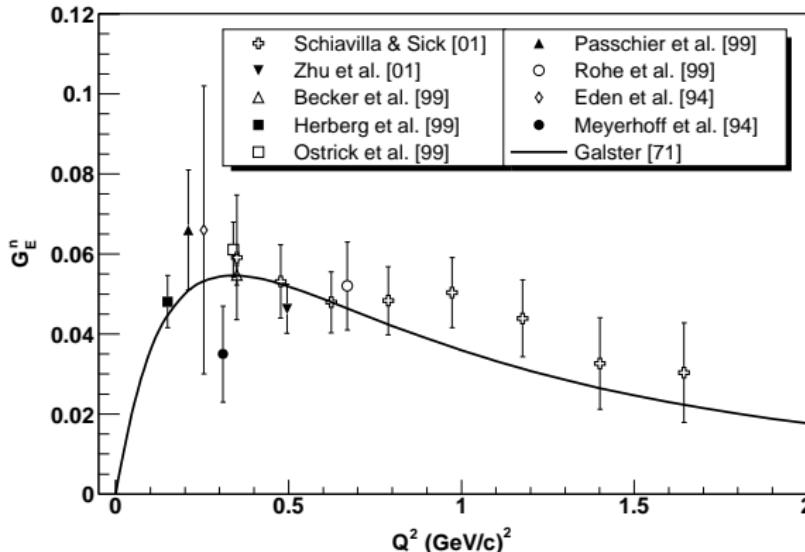
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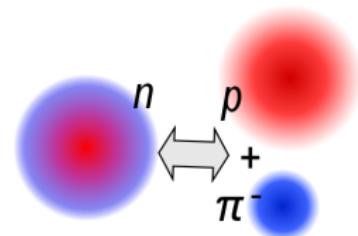
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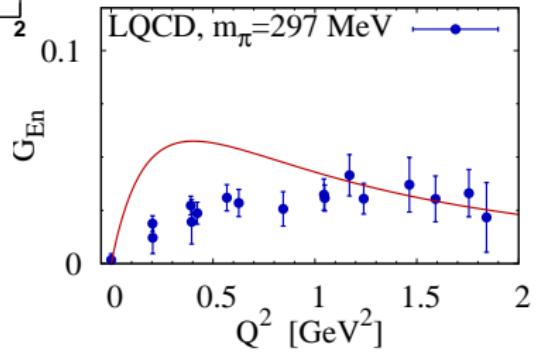


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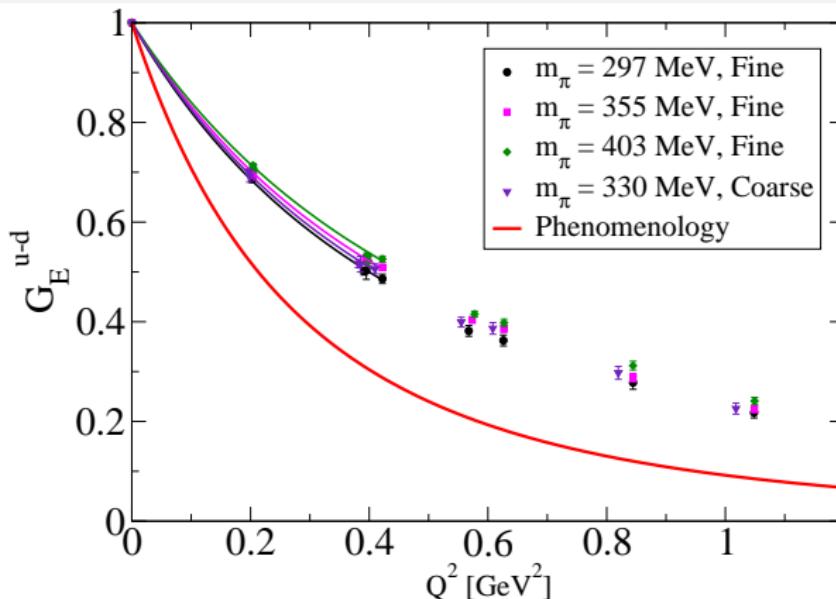
- deuterium targets
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PRL 74, 2427 (1995)



Isovector Electric Form Factor $G_E^{u-d} \equiv G_E^p - G_E^n$



Dipole fits

$$G \sim \frac{1}{(1+Q^2/M_D^2)^2}$$

give

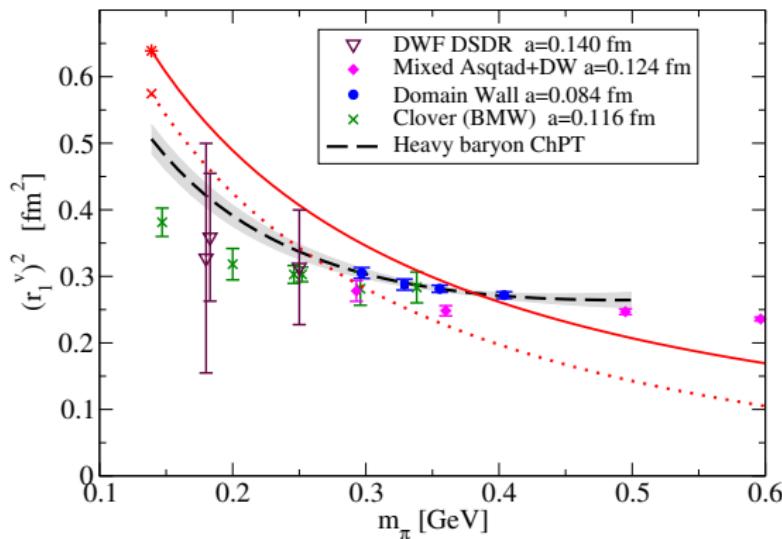
$$M_D^2 \approx 0.97 \dots 1.07 \text{ GeV}^2$$

$$(M_D^2)_{\text{exp}} = 0.71 \text{ GeV}^2$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \sim \langle N(+\frac{\vec{q}}{2}) | Q | N(-\frac{\vec{q}}{2}) \rangle$$

Fit to experimental data: [J. J. Kelly '04](#)

Isovector Dirac Radius $\langle r_1^2 \rangle^{u-d}$



Agreement for

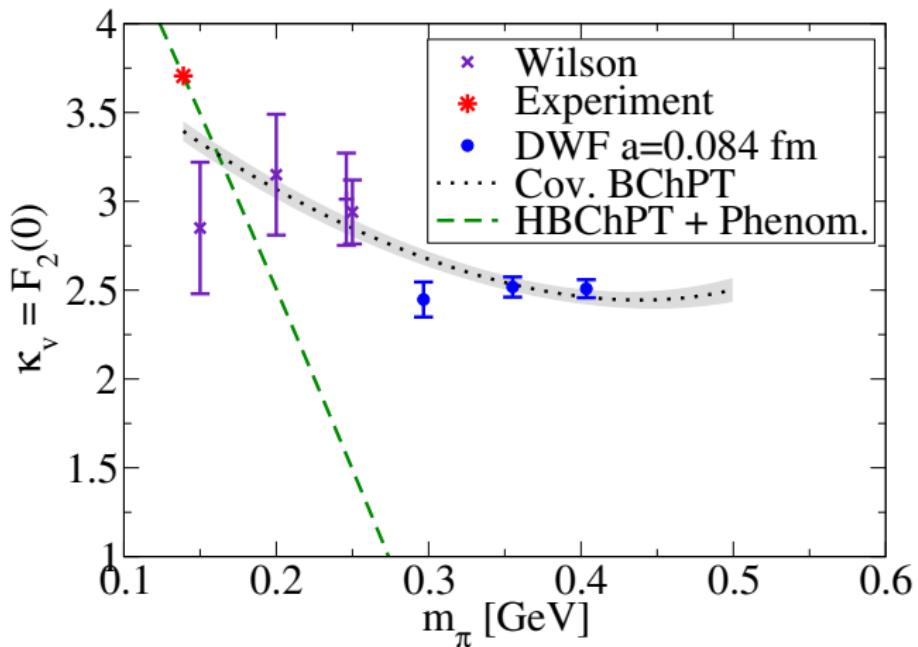
- 2 lattice spacings
- 2 volumes at $m_\pi = 250$ MeV
- 3 LQCD actions

pion loop contribution:
 $\delta[(r_1^2)^{u-d}] \sim \log m_\pi$
[V. Bernard et al '98](#)

Extract $\langle r_1^2 \rangle$ from dipole fits,

$$F_1(Q^2) \sim \frac{1}{(1 + \frac{1}{12} \langle r_1^2 \rangle Q^2)^2} \approx 1 - \frac{1}{6} \langle r_1^2 \rangle \cdot Q^2 + O(Q^4)$$

Anomalous Magnetic Moment κ_v



- Phenom. prediction: NLO HBChPT+ Δ [V. Bernard et al '98](#)
- Fit to lattice data: NLO CBChPT [T. Gail & T. Hemmert '08](#)

Nucleon Axial Form Factors

$$\langle N(p+q) | \bar{q} \gamma^\mu \gamma_5 q | N(p) \rangle = \bar{u}_{p+q} [\gamma^\mu \gamma_5 G_A + \frac{q^\mu}{2M} \gamma_5 G_P] u_p$$

with

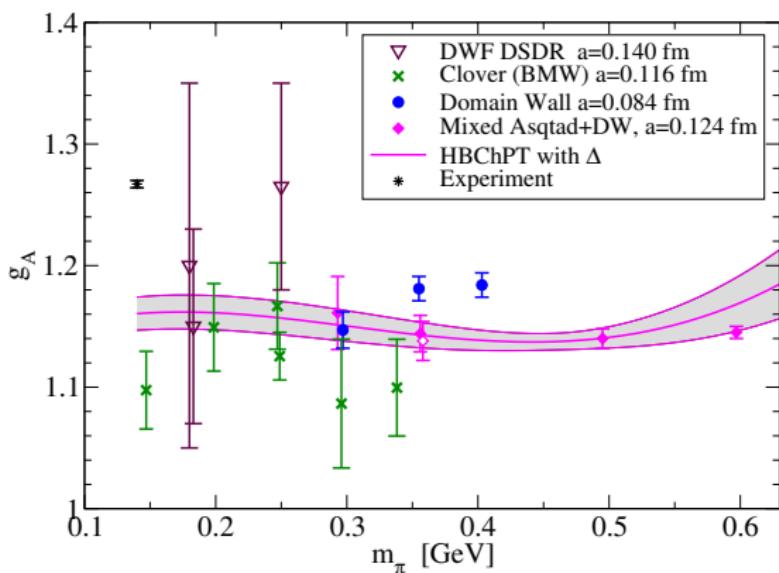
- $G_A(Q^2)$, axial form factor; axial charge $g_A = G_A(0)$
- $G_P(Q^2)$, induced pseudoscalar form factor

Nucleon structure seen by electroweak probes:

- $\nu, \bar{\nu}$ quasielastic scattering off protons & nuclei
- charged pion electroproduction $\gamma^* + N \rightarrow \pi^a + N'$
- muon capture $p + \mu \rightarrow n + \nu_\mu (+\gamma)$

Axial Charge g_A

- Computations are performed with $m_\pi \gtrsim 300$ MeV;
- need low-energy theory to extrapolate results to m_π^{phys}



Experiment $1.267(3)$

Fit to HBChPT + Δ

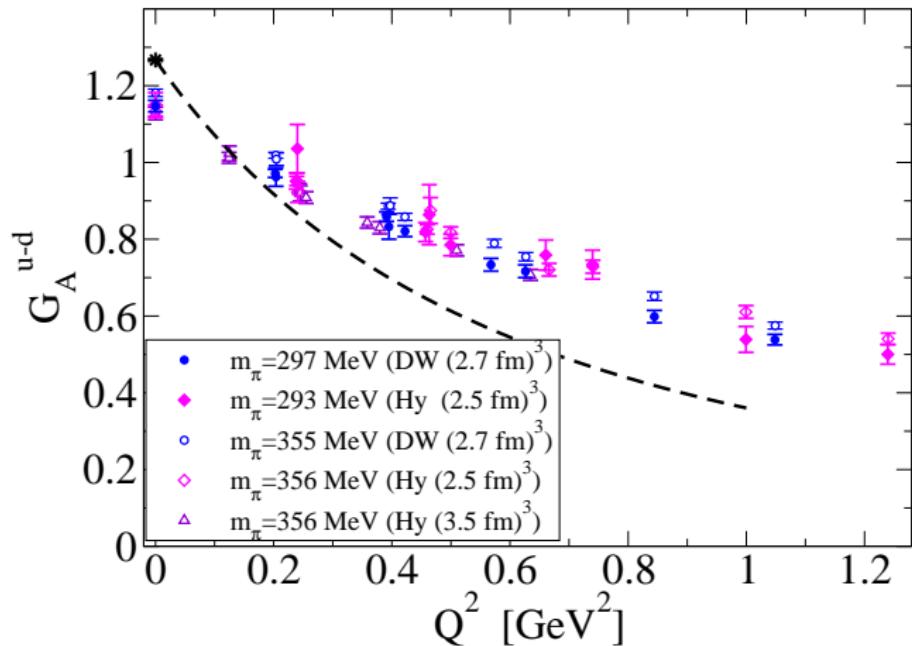
[T. Hemmert et al '03](#)

$1.160(14)$

$\approx 10\%$ smaller

- Finite-volume effects?
- Low-energy dynamics?

Axial Form Factor G_A^{u-d} vs. Q^2

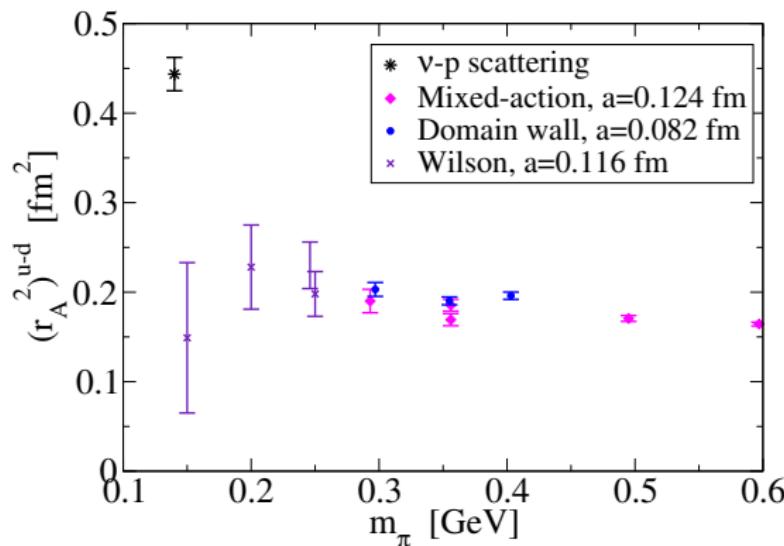


$$G_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2}$$

$$M_A^{\text{exp}} = (1.026 \pm 0.021) \text{ GeV } \text{PDG 2000}$$

$$M_A^{\text{lat}} \approx 1.5 \text{ GeV}$$

Axial Radius



$$\langle r_A^2 \rangle = -\frac{1}{6G_A} \frac{d G_A}{d Q^2} \Big|_{Q^2=0}$$

(dipole fits,
 $G_A \sim \frac{g_A}{(1+Q^2/M_A^2)^2}$)

ν -scattering:
 $\langle r_A^2 \rangle = (0.665(14) \text{ fm})^2$

NLO Heavy Baryon ChPT prediction: $\langle r_A^2 \rangle \approx \text{const}$

V. Bernard, H. Fearing, T. Hemmert, U.-G. Meissner (1998)

- rapid chiral dynamics beyond NLO ChPT?

Quark Momentum and Angular Momentum

Quark energy-momentum tensor $T_q^{\mu\nu}$

$$T_q^{\mu\nu} = \bar{q} \left[\gamma^{\{\mu} i \overset{\leftrightarrow}{D}^{\nu\}} - \langle \text{trace} \rangle \right] q$$

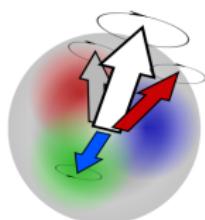
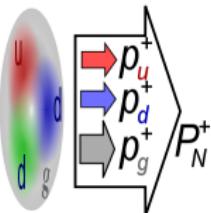
$$\langle N(P') | T_q^{\mu\nu} | N(P) \rangle \longrightarrow \{A_{20}, B_{20}, C_2\}(Q^2)$$

- quark momentum fraction

$$\langle x \rangle_q = A_{20}^q(0)$$

- quark angular momentum [X. Ji '97](#):

$$J_q = \frac{1}{2} [A_{20}^q(0) + B_{20}^q(0)]$$



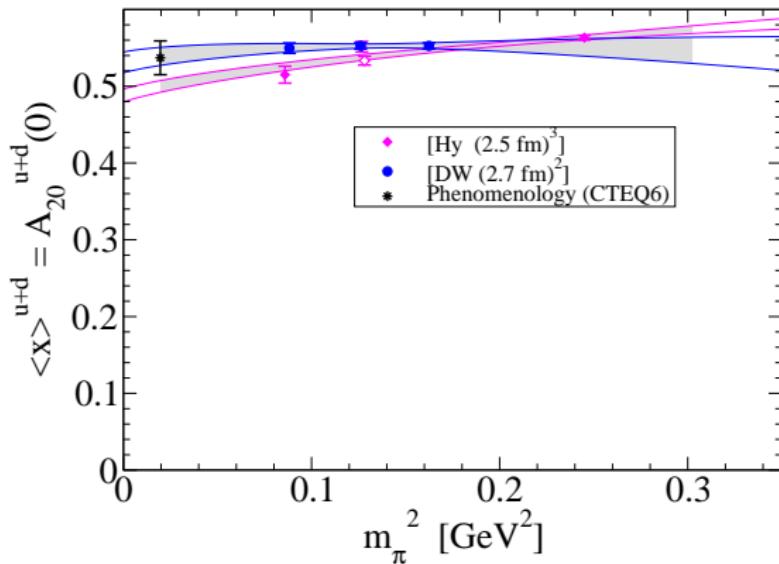
Separating contributions to nucleon spin:

- quark spin $S_q = \frac{1}{2} \Sigma_q = \frac{1}{2} \langle 1 \rangle_{\Delta q}$

- quark orbital angular momentum $L_q = J_q - S_q$

- gluons : the rest $J_{\text{glue}} = \frac{1}{2} - S_q - L_q$

Quark Momentum in the Proton $\langle x \rangle_{u+d}$



Boosted proton:

$$x_q = p_q^+ / P_p^+$$

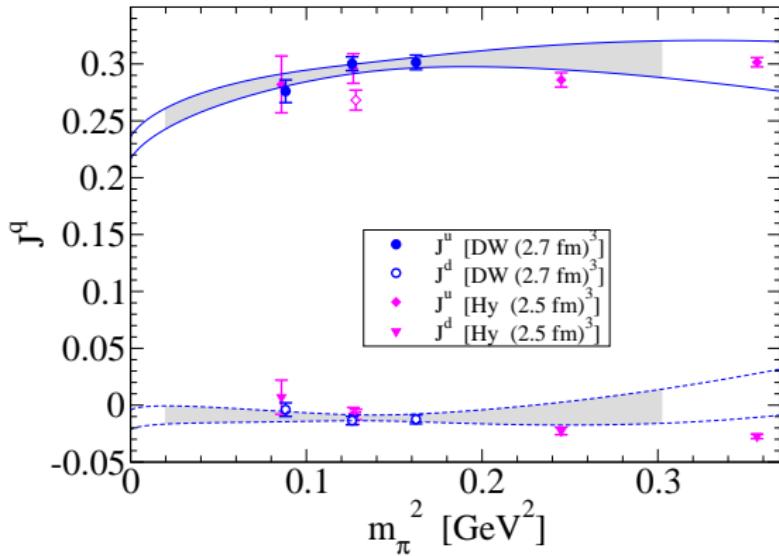
$$\langle x \rangle_q = A_{20}^q(0)$$

$$= \int dx x [q(x) + \bar{q}(x)]$$

Renormalized
to $\overline{\text{MS}}(2 \text{ GeV})$

- quarks carry $\approx 1/2$ of boosted nucleon momentum
- qualitative agreement with phenomenology

Quarks Angular Momentum: J^u, J^d



Following [X. Ji PRL '97](#),

$$J_q^3 = \langle N | \int d^3x M^{012} | N \rangle,$$

$$M^{\alpha\mu\nu} = x^\mu T_q^{\alpha\nu} - x^\nu T_q^{\alpha\mu}$$

and

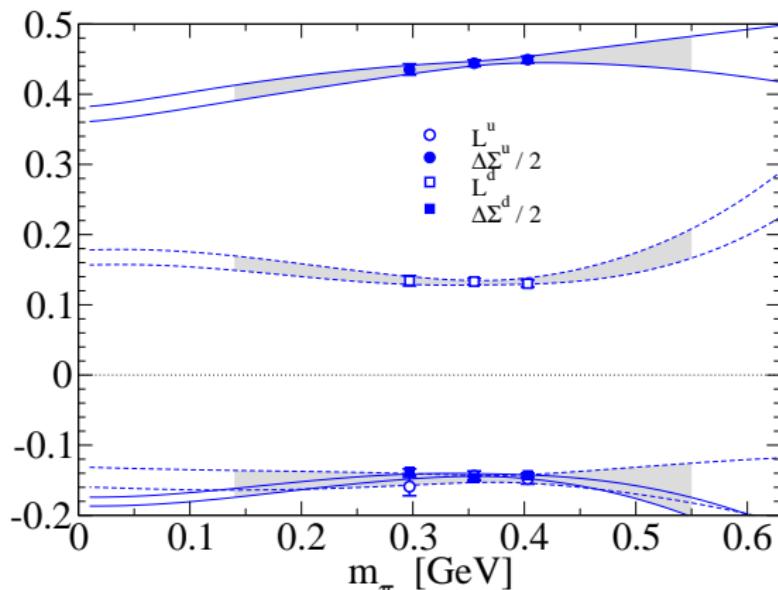
$$J^q = \frac{1}{2} [A_{20}^q(0) + B_{20}^q(0)]$$

Renormalized
to $\overline{\text{MS}}(2 \text{ GeV})$

Most contribution to the nucleon spine comes from u -quarks:

$$|J^d| \ll |J^u|$$

Quark Spin and OAM

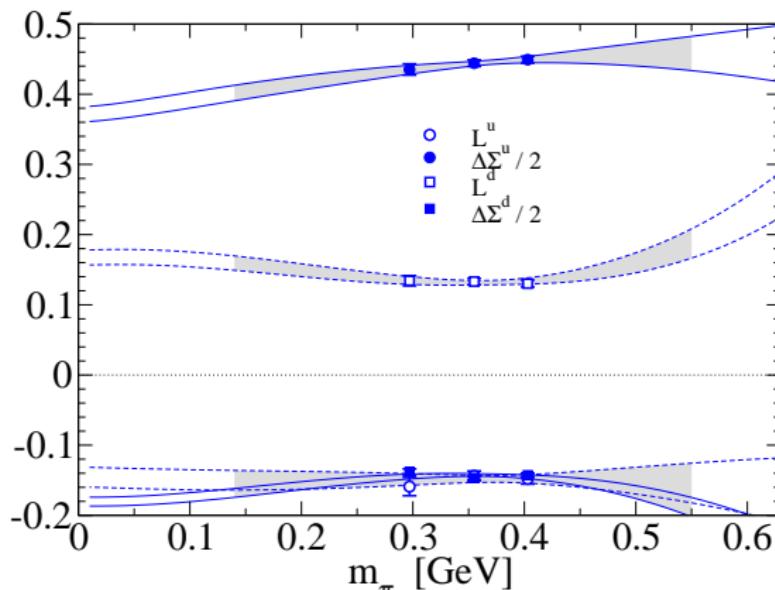


$$|J^d| \ll |S^d|, |L^d|, \quad (L^d + S^d \text{ cancel})$$

$$|L^{u+d}| \ll |L^u|, |L^d|, \quad (L^u + L^d \text{ cancel})$$

In agreement with Hägler *et al* (2007)

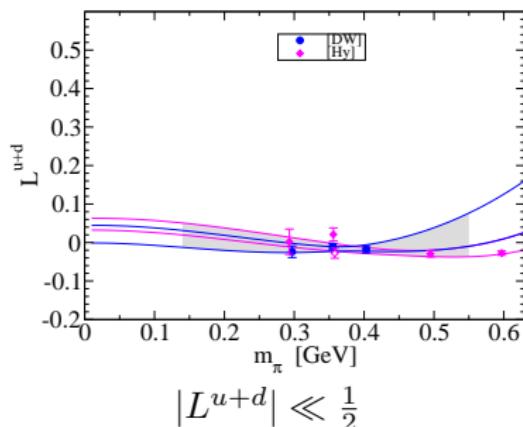
Quark Spin and OAM



$$\begin{aligned} |J^d| \ll |S^d|, |L^d|, \quad & (L^d + S^d \text{ cancel}) \\ |L^{u+d}| \ll |L^u|, |L^d|, \quad & (L^u + L^d \text{ cancel}) \end{aligned}$$

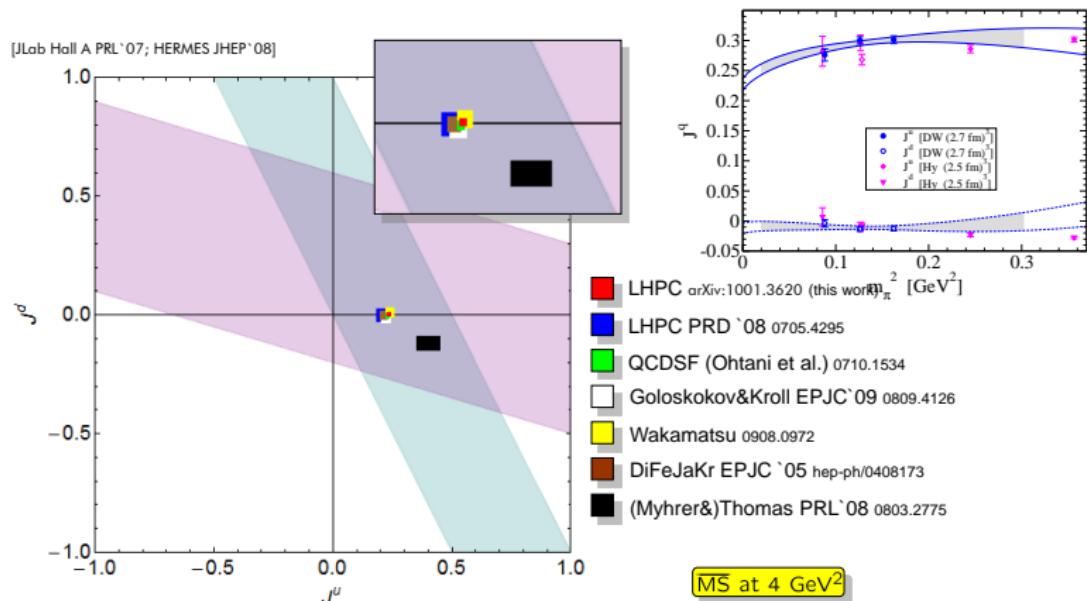
In agreement with Hägler *et al* (2007)

$$\begin{aligned} L^q &= J^q - S^q, \\ S_q &= \frac{1}{2} \Delta\Sigma_q \\ &= \int dx (\Delta q(x) + \Delta \bar{q}(x)) \end{aligned}$$



$$|L^{u+d}| \ll \frac{1}{2}$$

Quark Angular Momentum: p,n-DVCS



Ph. Hägler, MENU 2010, W&M

1

- DVCS provides GPD values $\{\mathcal{H}, \mathcal{E}\}(x, \xi, t)$ at $x = \pm \xi$
- Exp.values $J^{u,d} = \frac{1}{2} \int dx x (\mathcal{H} + \mathcal{E}) \Big|_{\xi=0, t=0}$ are model-dependent

1 Introduction

2 Nucleon Structure from Lattice QCD

- Electromagnetic Form Factors
- Axial Form Factors
- Quark momentum and angular momentum

3 Summary

Summary & Outlook

- Lattice simulations of non-perturbative QCD are necessary to understand the hadron structure
- “Nucleon size” observables (e.g. Dirac radius $\langle r_{1,2}^2 \rangle^v$) *undershoot* experimental values and weakly depend on m_π
- Quark contributions to nucleon momentum and spin are *in qualitative agreement* with phenomenology
- LQCD calculations of nucleon structure are still in “validation stage”: attempting to reproduce numbers known from experiment/phenomenology
- New generation of machines (PetaFLOP range) should be able to achieve physical m_π with good control over systematic uncertainties